

## 2HDM in terms of observable quantities

I. F. Ginzburg, K. A. Kanishev

*Sobolev Institute of Mathematics, Novosibirsk, 630090,  
Russia; Novosibirsk State University, Novosibirsk, 630090, Russia*  
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We found a minimal and a comprehensive set of directly measurable quantities defining the most general Two-Higgs-Doublet Model (2HDM), we call these quantities *observables*. The parameters of potential of the model are expressed explicitly via these observables (plus non-physical parameters similar gauge parameters). The model with arbitrary values of these observables can in principle be realized (up to general enough limitations). Our results open the door for the study of Higgs models in terms of measurable quantities only. The experimental limitations can be implemented here directly, without complex, often model-dependent, analysis of the Lagrangian coefficients.

The principal opportunity to determine all parameters of 2HDM from the (future) data meets strong practical limitation. In the best case it is the problem for very long time.

Apart from this construction *per se*, we also obtain some by-products. Among them – a simple criterium for CP conservation in the 2HDM, a new sum rules for Higgs couplings, a clear possibility of the coexistence of relatively light Higgses with the strong interaction in the Higgs sector and a simple expression for the triple Higgs vertex  $g(h_a h_a h_a)$ , useful for the analysis of future  $hhh$  coupling measurements.

### I. INTRODUCTION

The recent discovery of a Higgs-like particle with  $M \approx 125$  GeV at the LHC [1] hints that the spontaneous electroweak symmetry breaking is most probably realized by the Higgs mechanism. The minimal realization of the Higgs mechanism introduces a single scalar isodoublet  $\phi$  with the Higgs potential  $V_H = -m^2(\phi^\dagger \phi)/2 + \lambda(\phi^\dagger \phi)^2/2$ . This model is usually called “the Standard Model” (SM); it can be also referred as the “one Higgs doublet model”. The experimental results favor the realization of that minimal scenario [2] (*SM-like scenario* [3], or *SM alignment limit* [4]). Nevertheless, many variants of extended Higgs models are not ruled out.

The Two Higgs Doublet Model (2HDM) presents the simplest extension of the standard Higgs mechanism. This name unites a group of models in which standard Higgs doublet is supplemented by an extra hypercharge-one doublet. It offers a number of phenomenological scenarios with different physical content realized in different regions of the model parameter space, such as a natural mechanism for spontaneous CP violation [5]. Some of its variants have a number of interesting cosmological consequences [6]. The Higgs sector of the MSSM is a particular case of 2HDM, etc. Below we use the term 2HDM for the most general Two Higgs Doublet Model – independently on possible CP violation, violation of  $Z_2$  symmetry and with arbitrary Yukawa sector, etc.

After EWSB the 2HDM contains 3 neutral Higgs bosons  $h_a \equiv h_{1,2,3}$  and charged Higgs boson  $H^\pm$  with masses  $M_a$ ,  $M_\pm$  respectively.

In the SM, parameters of Higgs potential can be treated as measurable quantities, the mass of the Higgs boson  $M_h$  and the Higgs self-coupling parameter  $\lambda = M_h^2/v^2$ , where  $v = 246$  GeV is vacuum expectation value of Higgs field. Physical problems in this model

can be equally discussed in terms of parameters of the potential or in terms of these observables.

2HDM contains two fields with identical quantum numbers. Therefore, its description in terms of original fields or in terms of their linear superpositions are equivalent; this statement verbalizes the reparameterization (RPa) freedom of the model. A restricted version of this freedom, in which one only changes the phases of individual doublets, is called the rephasing (RPh) freedom. This freedom makes clear that the study in terms of Lagrangian may be likened to discussion of electrodynamical effects in a certain gauge defined by some particular gauge-fixing conditions. The discussion of 2HDM in terms of only well measurable quantities seems preferable.

The approach of many authors is to express measurable quantities via parameters of Lagrangian (in some RPa basis, often – with some simplifying assumptions) and to analyse physical phenomena via these RPa dependent parameters<sup>1</sup> (see e.g. [7], [8]). The basis-independent approach of [9] is often free from simplifying assumptions but involves very bulky calculations. The algebraic approach based on bilinears developed in [10] allows one to deduce general properties of the models but has no advantages for analysis of phenomenology.

- Our approach is opposite. We have developed a method for finding the minimal and a comprehensive set of directly measurable quantities defining the 2HDM and have built simple example of such set. Further we call these quantities *observables* and call the chosen complete set *The basic set of observables*. This basic set is

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<sup>1</sup> In the analysis of data the parameters of Higgs Lagrangian are being obtained by a solution of complex enough set of equations. The problem of completeness, or predetermination of the observations that are necessary for complete description of the potential is usually not discussed.

subdivided naturally into two subsets, defined below. We have found simple explicit expressions for the parameters of potential of the model via these observables (and non-physical parameters, fixing RPa basis). (In this calculation we continue the earlier studies of ref. [11].) This approach allows to analyse physical phenomena only in terms of measurable quantities, without using of RPa dependent parameters of potential. Fortunately, the obtained description appeared to be simple enough.

- The structure of the paper is the following. In Sec. II we reproduce well known facts in the useful for us form. We start with a brief review of the 2HDM and introduce useful notations. Next we turn to the Higgs basis, in which only one Higgs field have non-zero v.e.v. and which proves to be very useful for our analysis. In Sec. III we consider quadratic (mass) terms of the potential. We start with the neutral components of the original Higgs doublets and construct the physical Higgs fields. In the Higgs basis, the components of the corresponding rotation matrix represent measurable physical quantities. As a result, we express some of the parameters of the potential via the observables of the first subset – Higgs boson masses and their couplings to  $W^\pm$ . The study of triple and quartic interactions of Higgs fields is necessary to describe the remaining parameters of potential. It is done in Sec. IV, where we choose measurable vertices  $g(H^+H^-h_a)$  and  $g(H^+H^-H^+H^-)$  for the second subset of observables. In Sec. V, we consider Yukawa couplings for each type of fermions. In the Sec. VI we present some applications of obtained results and a number of useful by-products. Among them

- ▽ a simple criterium for CP conservation in the 2HDM, written via measurable quantities only;
- ▽ a new sum rules for Higgs couplings;
- ▽ a simple expression for the triple Higgs vertex  $g(h_a h_a h_a)$ , useful for the analysis of future  $hhh$  coupling measurements (in the separate paper);
- ▽ we have found the opportunity for co-existence of the relatively light Higgses and the strong interaction in the Higgs sector.

In Sec. VII we discuss the results obtained.

## II. TWO HIGGS DOUBLET MODEL

The 2HDM describes a system of two scalar isospinor fields  $\phi_1, \phi_2$  with hypercharge  $Y = 1$ . The most general form of the 2HDM potential is

$$\begin{aligned} V = & \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\ & + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{\lambda_5}{2} (\phi_1^\dagger \phi_2)^2 + \frac{\lambda_5^*}{2} (\phi_2^\dagger \phi_1)^2 \\ & + [\lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + \text{h.c.}] \\ & - \frac{m_{11}^2}{2} (\phi_1^\dagger \phi_1) - \frac{m_{22}^2}{2} (\phi_2^\dagger \phi_2) - \left[ \frac{m_{12}^2}{2} (\phi_1^\dagger \phi_2) + \text{h.c.} \right] \end{aligned} \quad (1)$$

Its coefficients are restricted by the requirement that the potential be positive at large quasiclassical values of  $\phi_i$  (*positivity constraints*).

- The model contains two doublets of scalar fields with identical quantum numbers. Therefore, it can be described either in terms of the original fields  $\phi_1, \phi_2$ , which enter (1), or in terms of fields  $\phi'_1, \phi'_2$ , which are obtained from  $\phi_k$  by a global unitary *reparameterization* transformation  $\hat{\mathcal{F}}$  of the form:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \hat{\mathcal{F}}_{gen}(\theta, \tau, \rho) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad (2)$$

$$\hat{\mathcal{F}}_{gen} = e^{-i\rho_0} \begin{pmatrix} \cos \theta e^{i\rho/2} & \sin \theta e^{i(\tau-\rho/2)} \\ -\sin \theta e^{-i(\tau-\rho/2)} & \cos \theta e^{-i\rho/2} \end{pmatrix}.$$

This transformation induces a transformation of the parameters of the Lagrangian  $\lambda_i \rightarrow \lambda'_i$  in such a way that the new Lagrangian, written in fields  $\phi'_i$ , describes the same physical content. We refer to these different choices as different RPa bases.

Transformation (2) is parameterized by angles  $\theta, \rho, \tau$  and  $\rho_0$ . The parameter  $\rho_0$  describes an overall phase transformation of the fields, and since it does not affect the parameters of the potential, we do not consider this degree of freedom.

In the potential (1), parameters  $\lambda_{1-4}, m_{11}^2$  and  $m_{22}^2$  are real while  $\lambda_{5-7}, m_{12}^2$  are generally complex. So, it takes 14 real quantities to fully define the scalar part of 2HDM. Since the three remaining parameters of RPa transformation cannot influence description of physical phenomena, the actual number of physically relevant parameters of the potential is  $14 - 3 = 11$ .

- Extrema of the potential satisfy the stationarity equations  $\partial V / \partial \phi_i|_{\phi_1=\langle\phi_1\rangle, \phi_2=\langle\phi_2\rangle} = 0$  ( $i = 1, 2$ ). The most general solution that describes the  $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$  symmetry breaking can be expressed via two positive numbers  $v_i$  and the relative phase factor  $e^{i\xi}$  as:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \quad (3)$$

$$v_1 = v \cos \beta, \quad v_2 = v \sin \beta, \quad v = \sqrt{v_1^2 + v_2^2}. \quad (4)$$

The ground state of potential (the vacuum) is the extremum with the lowest energy, and its vacuum expectation value (v.e.v.) is  $v = 246$  GeV.  $\phi_i$  are then decomposed into their v.e.v.'s and the quantized component fields:

$$\phi_1 = \begin{pmatrix} \xi_1^+ \\ v_1 + \zeta_1 + i\xi_1 \end{pmatrix} \frac{1}{\sqrt{2}}, \quad \phi_2 = \begin{pmatrix} \xi_2^+ \\ v_2 + \zeta_2 + i\xi_2 \end{pmatrix} \frac{1}{\sqrt{2}} e^{i\xi}. \quad (5)$$

Here,  $G^\pm = \xi_1^\pm \cos \beta + \xi_2^\pm \sin \beta$  and  $G^0 = \xi_1 \cos \beta + \xi_2 \sin \beta$  are the massless Goldstone modes, while  $H^\pm = \xi_2^\pm \cos \beta - \xi_1^\pm \sin \beta$  and  $\zeta_3 = \xi_2 \cos \beta - \xi_1 \sin \beta$  describe the charged Higgs boson and a neutral scalar

$\zeta_3$  whose parity is opposite to that of  $\zeta_{1,2}$ . Linear combinations of neutral fields  $\zeta_i$  form the set of observable neutral Higgs particles  $h_1, h_2, h_3$ .

• **Relative couplings.** Let us denote the coupling of each neutral Higgs boson to a fundamental particle  $P$  by  $g_a^P$  ( $P = \{V(W, Z), f = q(t, b, c, \dots), \ell(\tau, \mu, e)\}$ ) and these very couplings of the standard Higgs boson of SM as  $g_{\text{SM}}^P$ . Below, we make use of the relative couplings, the ratios of these couplings:

$$\chi_a^P = g_a^P / g_{\text{SM}}^P. \quad (6)$$

The model contains an extra scalar-vector boson interactions,  $H^\pm W^\mp h_a$  and interactions  $H^+ H^- h_a$ . For these we introduce dimensionless relative couplings:

$$\chi_a^{H^+ W^-} = \frac{g(H^+ W^- h_a)}{M_W / v} \equiv (\chi_a^{H^- W^+})^*; \quad (7)$$

$$\chi_a^\pm = g(H^+ H^- h_a) / (2M_\pm^2 / v). \quad (8)$$

The neutrals  $h_a$  generally have no definite CP parity. Couplings  $\chi_a^V$  and  $\chi_a^\pm$  are real due to Hermiticity of Lagrangian, while other couplings are generally complex.

Relations among relative couplings appearing in particular models are more stable under radiative corrections than the relations among couplings themselves, since possible large QCD corrections in nominator and denominator of (6) compensate each other.

We omit the adjective "relative" further in the text.

• **Higgs basis.** Any RPa basis can be used for solving physical problem. Some of them are more suitable than others when solving specific problems. In particular, when the system possesses an additional symmetry, the preferable RPa basis is the one in which this symmetry is made obvious. Examples include the case when, in some RPa basis, the Yukawa sector has a form in which fermions of each type are coupled to only one field  $\phi_1$  or  $\phi_2$  (well known models I, II, X, Y); the case of softly broken  $Z_2$  symmetry (the RPa basis with  $\lambda_6 = \lambda_7 = 0$ ); the explicitly CP symmetric model (the RPa basis with all parameters of the potential real), etc.

We find useful for our goals to analyze the model with known vacuum (the ground state of the potential) using the basis with  $v_2 = 0$ . This basis is called *the Higgs (or Georgi) basis* [13]. This basis is obtained from any given basis with known v.e.v.'s (4) by transformation (2) with

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \hat{\mathcal{F}}_{HB} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \hat{\mathcal{F}}_{HB} = \hat{\mathcal{F}}_{\text{gen}}(\theta = \beta, \tau = \rho - \xi). \quad (9)$$

The phase factor  $e^{\pm i\rho/2}$  represents the remaining RPh freedom in the choice of the Higgs basis that is, independence of the physical picture from the choice of relative phase  $\phi_i$ , the RPh phase.

*Vise versa*, any form of the potential can be obtained from the Higgs basis form with the transformation,  $\hat{\mathcal{F}}_{HB}^{-1} = \hat{\mathcal{F}}_{\text{gen}}(\theta = -\beta, \tau = \rho + \xi)$  with  $\rho \rightarrow -\rho, \rho_0 \rightarrow -\rho_0$ . We denote this basis as "RPa basis  $(\beta, \xi)/HB$ ". Again,

we do not fix in this definition the RPh phase  $\rho$  and the irrelevant parameter  $\rho_0$ .

The potential obtained has the same form as (1). To distinguish its parameters in the Higgs basis from a generic basis, we use the capital letters  $\Lambda$  for the quartic and  $\mu$  for the quadratic parameters. The extremum conditions in the Higgs basis are simple,  $v^2 \Lambda_1 = \mu_{11}^2$ ,  $v^2 \Lambda_6 = \mu_{12}^2$ . With these constraints the potential can be rewritten, up to a constant, in a more elegant form via the charged Higgs mass  $M_\pm^2$  [14]

$$\begin{aligned} V_{HB} = & M_\pm^2 \left( \Phi_2^\dagger \Phi_2 \right) + \frac{\Lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 - \frac{v^2}{2} \right)^2 + \frac{\Lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 \\ & + \Lambda_3 \left( \Phi_1^\dagger \Phi_1 - \frac{v^2}{2} \right) \left( \Phi_2^\dagger \Phi_2 \right) + \Lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) \\ & + \left[ \frac{\Lambda_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \Lambda_6 \left( \Phi_1^\dagger \Phi_1 - \frac{v^2}{2} \right) \left( \Phi_1^\dagger \Phi_2 \right) \right. \\ & \left. + \Lambda_7 \left( \Phi_2^\dagger \Phi_2 \right) \left( \Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right]. \end{aligned} \quad (10)$$

This fixing the RPa basis reduces the number of parameters needed to describe potential comparing to the original form (1). Instead of the four mass term parameters in its last line, we have two parameters, the v.e.v.  $v = 246$  GeV and the mass of charged Higgs boson  $M_\pm$ . The quartic part still contains 10 dimensionless parameters  $\Lambda_i$  (including real and imaginary parts of  $\Lambda_{5,6,7}$ ). The residual RPh freedom is described with the irrelevant basis parameter  $\rho$ , the relative phase between the fields  $\phi_i$ . The total number of relevant free parameters is 11, as mentioned above.

In the Higgs basis, the decomposition (5) simplifies to

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{v + \eta_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{\eta_2 + i\eta_3}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

To arrive to the description in terms of physically observable fields, one should start by substituting these expressions into the potential (10). Also, by choosing the unitarity gauge for the gauge fields, we omit the Goldstone modes  $G^a$  from now on.

As a result, the potential (10) is takes the form in which coefficients are expressed via parameters of (10) (here and below, the usual convention of summation over repeated indices is adopted):

$$\begin{aligned} V = & M_\pm^2 H^+ H^- + \frac{M_{ij}}{2} \eta_i \eta_j \\ & + v T_i H^+ H^- \eta_i + v T_{ijk} \eta_i \eta_j \eta_k \\ & + C H^+ H^- H^+ H^- \\ & + \frac{B_{ij}}{2} H^+ H^- \eta_i \eta_j + Q_{ijkl} \eta_i \eta_j \eta_k \eta_l. \end{aligned} \quad (12)$$

### III. QUADRATIC TERMS OF POTENTIAL (12). FIRST GROUP OF OBSERVABLES

In eq. (12), the coefficients  $M_{ij}$  form the neutral scalar mass matrix (here  $N = M_{\pm}^2/v^2 + \Lambda_4$ ):

$$M_{ij} = v^2 \begin{pmatrix} \Lambda_1 & \frac{Re \Lambda_6}{2} & -Im \Lambda_6 \\ Re \Lambda_6 & \frac{N + Re \Lambda_5}{2} & -Im \Lambda_5/2 \\ -Im \Lambda_6 & -Im \Lambda_5/2 & \frac{N - Re \Lambda_5}{2} \end{pmatrix}. \quad (13)$$

The physical neutral Higgs states  $h_a$  are such superpositions of fields  $\eta_i$  that diagonalize this mass matrix:

$$h_a = R_a^i \eta_i, \quad \eta_i = R_i^a h_a; \quad (14)$$

$$M_{ij} \eta_i \eta_j / 2 = \sum_a M_a^2 h_a^2 / 2, \quad M_{ij} = R_i^a R_j^a M_a^2. \quad (15)$$

The mixing matrix  $R_i^a$  is a real-valued orthogonal matrix determined by the parameters of the mass matrix. It can be parameterized with three Euler angles. One of them is responsible for rephasing transformation of fields, i.e. it is irrelevant. The overall sign of this matrix is insignificant, we fix  $R_1^1 > 0$ .

The trace of the mass matrix is invariant under orthogonal transformations (14). Therefore we obtain a sum rule:

$$v^2 (\Lambda_1 + \Lambda_4) = \sum_a M_a^2 - M_{\pm}^2. \quad (16)$$

One of the advantages of the Higgs basis as compared to other RPa bases is the fact that elements of rotation matrix are directly related to the couplings (6), (7), which are, in principle, measurable:

$$\chi_a^V = R_1^a, \quad \chi_a^{H^+W^-} \equiv \left( \chi_a^{H^-W^+} \right)^* = R_2^a + i R_3^a. \quad (17)$$

It can be seen easily after writing the kinetic term of Higgs Lagrangian with definitions (11), (14). The absolute values of the real quantities  $\chi_a^V$  are directly measurable in the decays  $h_a \rightarrow WW$  (or  $W$ -fusion process), etc.

The phases of quantities  $\chi_a^{H^+W^-}$ , i.e. the ratios  $R_3^a/R_2^a$  cannot be fixed because of the rephasing freedom of potential in the Higgs basis. Their relative phases for different  $h_a$  are determined unambiguously since they describe the physical quantity  $\chi_{ab}^Z$  (33). In particular, one can fix the rephasing phase (i.e. the RPh basis) by the condition  $R_3^2 = 0$  (or some other condition).

The orthogonality of the mixing matrix means that its elements obey a set of relations:

$$\sum_i R_i^a R_i^b = \delta_{ab}, \quad \sum_a R_i^a R_j^a = \delta_{ij}. \quad (18)$$

In a RPh basis with  $R_3^2 = 0$ , the relations (18) with  $a = b$  and  $i = j$  can be treated as equations determining some elements of mixing matrix via the others. Namely, we

express all elements  $R_i^a$  via couplings of different Higgs neutrals  $h_a$  to gauge bosons  $\chi_a^V$ . The relative signs of some elements of this matrix are given by basic expression of mixing matrix via Euler angles. Including the phase rotation and restoring thus the rephasing freedom to allow for the phase  $\rho$  in  $\chi_2^{H^+W^-}$ , we write general form of the mixing matrix:

$$R_a^i = \begin{pmatrix} \chi_1^V & \chi_2^V & \chi_3^V \\ \frac{-\chi_1^V \chi_2^V}{\sqrt{1-(\chi_2^V)^2}} & \sqrt{1-(\chi_2^V)^2} & \frac{-\chi_2^V \chi_3^V}{\sqrt{1-(\chi_2^V)^2}} \\ \frac{\chi_3^V}{\sqrt{1-(\chi_2^V)^2}} & 0 & \frac{-\chi_1^V}{\sqrt{1-(\chi_2^V)^2}} \end{pmatrix} T, \quad (19)$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \rho & \sin \rho \\ 0 & -\sin \rho & \cos \rho \end{pmatrix}$$

with limitation, given by sum rule

$$\sum_a (\chi_a^V)^2 = 1. \quad (20)$$

Finally, one can read (13) as expressions of some  $\Lambda$ 's via elements of the mass matrix and then, with the aid (15), to express them via the masses of Higgs bosons and their couplings to gauge bosons (for  $\Lambda_4$  we prefer to use the sum rule (16)):

$$\begin{aligned} v^2 \Lambda_1 &= \sum_a (\chi_a^V)^2 M_a^2; \\ v^2 \Lambda_4 &= \sum_a M_a^2 - M_{\pm}^2 - v^2 \Lambda_1; \\ v^2 \Lambda_5^* &= \sum_a (\chi_a^{H^+W^-})^2 M_a^2; \\ v^2 \Lambda_6^* &= \sum_a \chi_a^V \chi_a^{H^+W^-} M_a^2. \end{aligned} \quad (21)$$

The observables entering into this equation form a first subset of the basic set of observables. Couplings  $\chi_a^{H^+W^-}$  are expressed via  $\chi_a^V$  by eq. (19). The phase freedom in the definition of these couplings is reproduced as a similar freedom in phases of  $\Lambda_5, \Lambda_6$ .

### IV. OTHER TERMS OF POTENTIAL

The Higgs bosons masses and couplings to the gauge bosons do not depend on  $\Lambda_2, \Lambda_3, \Lambda_7$ . In turn, these parameters are necessary to determine triple and quartic Higgs bosons vertices. The measurements of some of these vertices are necessary to obtain complete set of observables. To construct second subset of observables, supplementing first subset to the basic set of variables (minimal and comprehensive), we consider triple and quartic interactions with the charged Higgs boson. In each case we start our analysis in terms of fields  $\eta_i$  and then pass to the physical Higgs fields  $h_a$  with the aid of the mixing matrix (14).



### A. Cubic terms of potential (12)

Each triple Higgs vertex depends on  $\Lambda_3$ ,  $Re\Lambda_7$ ,  $Im\Lambda_7$ , in addition to the parameters of the first subset.

- We use for basic set the **vertices**  $H^+H^-h_a$ . The simplest part of the cubic terms in (12) describes interaction of neutral and charged scalars:

$$vT_i H^+H^- \eta_i, \quad \text{with} \quad T_i = (\Lambda_3, Re\Lambda_7, -Im\Lambda_7)_i. \quad (22)$$

After transformation to physical states  $\eta_i = R_i^a h_a$ , we obtain the corresponding couplings:

$$g(H^+H^-h_a) = vR_i^a T_i.$$

This expression is easy to solve for  $T_i$  by inverting the rotation matrix. In this way, we express three parameters  $\Lambda_3$ ,  $Re\Lambda_7$ ,  $Im\Lambda_7$  via the measurable couplings between the neutral and charged scalars (cf. notation (8)):

$$\begin{aligned} \Lambda_3 &= (2M_\pm^2/v^2) \sum_a \chi_a^V \chi_a^\pm; \\ \Lambda_7^* &= (2M_\pm^2/v^2) \sum_a \chi_a^{H^-W^+} \chi_a^\pm. \end{aligned} \quad (23)$$

- **Vertices**  $h_a h_b h_c$  arise from the part of potential (12), cubic in neutral scalars:

$$\begin{aligned} vT_{ijk} \eta_i \eta_j \eta_k &= \frac{v}{2} [\eta_1^3 \Lambda_1 + \eta_1 \eta_2^2 (\Lambda_3 + \Lambda_4 + Re\Lambda_5) \\ &\quad + \eta_1 \eta_3^2 (\Lambda_3 + \Lambda_4 - Re\Lambda_5) - 2\eta_1 \eta_2 \eta_3 Im\Lambda_5 \\ &\quad + 3\eta_1^2 (\eta_2 Re\Lambda_6 - \eta_3 Im\Lambda_6) \\ &\quad + (\eta_2^3 + \eta_2 \eta_3^2) Re\Lambda_7 - (\eta_2^2 \eta_3 + \eta_3^3) Im\Lambda_7]. \end{aligned} \quad (24)$$

This equation contains parameters which are already familiar from quadratic terms and from eq. (23). Passing as usual to the physical fields  $h_a$ , we transform this term to a form which exposes the triple neutral Higgs interactions  $h_a h_b h_c$ .

In the important particular case  $b = c = a$ , the using the orthogonality relations (18) gives

$$\begin{aligned} g(h_a h_a h_a) &= \\ v [\Lambda_1 (R_1^a)^3 + (\Lambda_3 + \Lambda_4) R_1^a (1 - (R_1^a)^2) \\ &\quad + Re\Lambda_5 R_1^a ((R_2^a)^2 - (R_3^a)^2) - 2Im\Lambda_5 R_1^a R_2^a R_3^a \\ &\quad + 3(R_1^a)^2 (Re\Lambda_6 R_2^a - Im\Lambda_6 R_3^a) \\ &\quad + (1 - (R_1^a)^2) (Re\Lambda_7 R_2^a - Im\Lambda_7 R_3^a)]. \end{aligned} \quad (25)$$

### B. Quartic terms of potential (12)

The parameter  $\Lambda_2$  can only be extracted from quartic couplings. Each quartic Higgs vertex depends on parameter  $\Lambda_2$  in addition to parameters determined from mass terms and triple Higgs couplings.

- We use for basic set **the vertex**  $H^+H^-H^+H^-$ . In Higgs basis, the charged Higgs bosons arise only from  $\Phi_2$  (11). Therefore, the  $H^+H^-H^+H^-$  vertex enters

Lagrangian in a very simple form  $\frac{\Lambda_2}{2} H^+H^-H^+H^-$ , and its observation offers the simplest way to measure  $\Lambda_2$ :

$$\Lambda_2 = 2g(H^+H^-H^+H^-). \quad (26)$$

- **The quartic scalar vertices involving neutrals**  $h_a$  are given by the sum  $\frac{B_{ij}}{2} H^+H^- \eta_i \eta_j + Q_{ijkl} \eta_i \eta_j \eta_k \eta_l$  in eq. (12). The coefficients here include the parameter  $\Lambda_2$ , for example

$$B_{ij} = \begin{pmatrix} \Lambda_3 & Re\Lambda_7 & -Im\Lambda_7 \\ Re\Lambda_7 & \Lambda_2 & 0 \\ -Im\Lambda_7 & 0 & \Lambda_2 \end{pmatrix} \quad (27)$$

Experimental measurement of these vertices gives a cross check for the value of  $\Lambda_2$  obtained from the  $H^+H^-H^+H^-$  interaction. The corresponding algebraic expressions are straightforward but cumbersome.

### C. Other possible choices

The second subset of observables can be constructed with other triple and quartic couplings.

The using of processes involving charged Higgses looks preferable for two reasons. First, with charged Higgses, this procedure requires the fewest calculations, improving accuracy and reducing uncertainties. Second, the amplitudes of the processes  $e^+e^- \rightarrow H^+H^-h_a$ ,  $\gamma\gamma \rightarrow H^+H^-h_a$ ,  $e^+e^- \rightarrow H^+H^-H^+H^-$ ,  $\gamma\gamma \rightarrow H^+H^-H^+H^-$  at ILC/CLIC [15] are directly proportional to the corresponding couplings, without any non-relevant diagrams interfering.

## V. YUKAWA INTERACTION

The 2HDM can admit different forms of the Yukawa sector. The interaction of a given right-handed down type fermion  $f$  to neutral components of the Higgs fields can be written, in the starting notation, as  $\Delta_f L_Y = \bar{f}_L (g_1 \Phi_1^0 + g_2 \Phi_2^0) f_R + h.c.$ , where  $\phi_i^0$  stand for the neutral component of  $\phi_i$ . A simple RPa transformation  $g_1 \Phi_1^0 + g_2 \Phi_2^0 = g_f \bar{\phi}_{1f}^0$  brings us to the  $f$ -selective RPa basis, which is adapted for this very fermion and in which the mass-generating Yukawa interaction is written as

$$\Delta_f L_Y = g_f \bar{f}_L \bar{\phi}_{1f}^0 f_R + h.c. \quad (28)$$

For a generic Yukawa sectors, these RPa bases are different for each fermion  $f$ . (For up-type quarks, the same expressions with  $\Phi_i^*$  are assumed.)

In this  $f$ -selective RPa basis, the v.e.v.'s of Higgs fields have form (4) with parameters  $\beta_f$ ,  $\xi_f$ , determined independently. This basis is obtained from the Higgs basis form by a transformation inverse to (9), i.e. that is  $(\beta_f, \xi_f)/HB$  RPa basis.

Fixing  $\rho_0 = \rho/2$ , we express the field  $\bar{\phi}_{1f}$  via the Higgs basis fields  $\Phi_i$  by equation  $\bar{\phi}_{1f} = \cos\beta_f\Phi_1 - \sin\beta_f e^{i\xi_f}\Phi_2$ . In this basis, the v.e.v. of field  $\bar{\phi}_{1f}$  is  $v\cos\beta_f$ , which allows us to compactly write the corresponding couplings; for example,  $g_\phi^t = g_{SM}^t/\cos\beta_t$ . Now, decomposition of neutral components of  $\Phi_i$  (11) gives the interaction of these neutral component to  $f$  in the form

$$\Delta_f L_Y = g_f^{SM} \frac{\bar{f}_L [\cos\beta_f \eta_1 + \sin\beta_f e^{i\xi_f} (\eta_2 + i\eta_3)] f_R + h.c.}{\cos\beta_f} \quad (29)$$

After that, the substitution of the rotation matrix (14) gives couplings of down  $f$ -quark to all neutral Higgses in the form

$$\chi_a^f = \chi_a^V + \tan\beta_f e^{i\xi_f} \chi_a^{H^+W^-}. \quad (30)$$

In the CP conserved case these equations can be easily transformed to well known forms.

(For the up quark  $f$  in the similar way one should write  $\phi_i^*$  instead of  $\phi_i$ , it results in to the change  $\xi \rightarrow -\xi$  in the final equations.)

It is known that a generic 2HDM Yukawa sector leads also to the flavor-changing neutral currents (FCNC). They arise due to mismatch of the  $f$ -selective bases of the quarks of the same charge. If needed, these FCNC couplings can also be expressed, in a similar fashion, via the mismatch angles.

- It is instructive to illustrate this discussion with some discussed Yukawa sectors.

In 2HDM-I (Yukawa Model I), the  $f$ -preferable bases coincide for all fermions,  $\beta_t = \beta_b$ ,  $\xi_t = \xi_b$ ,

$$\chi_a^u = \chi_a^d = \chi_a^\ell. \quad (31a)$$

In 2HDM-II (Yukawa Model II), one such basis describes all up-quarks, and an orthogonal basis describes all down quarks,  $\beta_b = \pi/2 - \beta_t \equiv \beta$ ,  $\xi_b = \xi$ ,  $\xi_t = 0$ , etc. It leads to useful relations among Yukawa couplings for different fermions [12]

$$(\chi_a^u + \chi_a^d)\chi_a^V = 1 + \chi_a^u\chi_a^d. \quad (31b)$$

In the aligned 2HDM [19], one has a similar picture — one  $f$ -selective basis for all up quarks, and another basis for all down quarks, — but these two bases are not assumed to be orthogonal, so that one can try to construct equations like (31b) having more complex form.

## VI. SOME APPLICATIONS AND BY-PRODUCTS

Our results allow to obtain a number of useful equations and to convert some known facts to a form more convenient for data analysis.

- **Positivity constraints, etc.** The positivity constraints can be now rewritten via the measurable

quantities from the basic set. For example, well known equation  $\sqrt{\Lambda_1\Lambda_2} + \Lambda_3 > 0$  is read now as

$$\sqrt{2g(H^+H^-H^+H^-)\sum_a(\chi_a^V)^2M_a^2v^2} + 2M_\pm^2\sum_a\chi_a^V\chi_a^\pm > 0. \quad (32)$$

Similar equations can be written for other positivity constraints and for the perturbativity and unitarity constraints, etc.

**II. Coupling  $Zh_a h_b$ .** The direct substitution of the rotation matrix into the kinetic term of Lagrangian gives coupling  $Zh_a h_b$  in the form

$$\chi_{ab}^Z \equiv \frac{g(Zh_a h_b)}{M_Z/v} = R_2^a R_3^b - R_2^b R_3^a.$$

This equation can be rewritten via couplings  $\chi_a^{H^\mp W^\pm}$  and then — with the aid of eq. (19) — in the very simple form:

$$\chi_{ab}^Z = \text{Im} \left( \chi_a^{H^\mp W^\pm} \chi_b^{H^\pm W^\mp} \right) \equiv -\varepsilon_{abc} \chi_c^V. \quad (33)$$

Here such equation, known for CP conserving case, is spread for the most general case.

### • III. Signature for CP conservation in 2HDM.

In general, the neutrals  $h_a$  have no definite CP parity. The condition for CP conservation in the model is written as a pair of almost obvious identities

$$\prod_a \chi_a^V = 0; \quad \prod_a \chi_a^\pm = 0. \quad (34a)$$

From Eq-s (21), (23) one can see that these identities allow to have potential in Higgs basis with all real coefficients. It means that identities (34a) form *necessary and sufficient conditions for CP conservation in 2HDM*. In this case number of independent basic observables is reduced by 2 — from 11 to 9, as it is known for the general explicitly CP-conserving case of 2HDM.

The using of eqs. (17), (19) and (30) shows that the first condition (34a) ensures the validity of the necessary condition of the CP conservation for each fermion

$$\left| \prod_a \chi_a^f \right| = \prod_a |\chi_a^f|. \quad (34b)$$

Let us note that even weak violation of CP for coupling  $h_a$  to gauge boson is compatible with strong enough violation of CP in the  $\bar{f}h_a f$  interaction (if corresponding  $\tan\beta_f$  (or  $\cot\beta_f$ ) is large).

**IV. Sum rules for Higgs couplings.** It is useful, in addition to (19), to rewrite equations (18) at  $a = b$  and at  $i = j$  and eq. (30) in the form of sum rules, allowing for generalizations to some other forms of the Higgs sector [16]. That are relation (20) and

$$(a) \sum_a (\chi_a^f)^2 = 1, \quad (b) |\chi_a^V|^2 + |\chi_a^{H^+W^-}|^2 = 1. \quad (35)$$

Relations (20) and (a) are the sum rules for couplings of different neutral scalars to gauge bosons and fermions. They are well known for the CP conserving case and for some definite forms of the Yukawa sector (see e.g. [17, 18]). The main new point here is the statement about validity of all these sum rules beyond CP conservation and for an arbitrary form of the Yukawa sector.

The relations (b) for  $a = 1, 2, 3$  represent a new set of sum rules, which is useful for assessing the physics potential of the forthcoming experiments.

**V. Possible strong interaction in the Higgs sector.** The fact that free parameters of the potential naturally fall into three very distinct categories, offers a new opportunity which was absent in the SM. Before the Higgs discovery, the large coupling constant  $\lambda$  was, in principle, possible within SM. In this case, the Higgs boson would be very heavy and wide, and it could not be seen as separate particle. Instead, its dynamics would be governed by the strong interaction in the Higgs sector, which would manifest itself in the form of resonances in the  $W_L W_L$ ,  $W_L Z_L$ ,  $Z_L Z_L$  scattering in the 1-2 TeV energy range. In the SM this opportunity is closed by the discovery of the Higgs boson with  $M \approx 125$  GeV.

Our analysis shows that, within 2HDM, the reasonably low values of all Higgs masses are well compatible with large  $\Lambda_3$ ,  $|\Lambda_7|$ ,  $\Lambda_2$ , i.e. with the strong interaction in the Higgs sector. A signal of this feature can be observed in the multi-Higgs final states or (for  $\Lambda_3$ ,  $|\Lambda_7|$ ) in anomalously large two-photon width of some neutral Higgs boson. Moreover, this strong interaction can coexist even with moderate values of triple Higgs couplings as it could be driven exclusively by the large value of a single parameter  $\Lambda_2$ .

**VI. Triple Higgs vertex.** To express triple Higgs vertex (25) in terms of observables we use (21) and (23). We obtain

$$\begin{aligned} g(h_a h_a h_a) &= (M_a^2/v) \chi_{aaa}; \\ \chi_{aaa} &= \chi_a^V \left\{ 1 + (1 - (\chi_a^V)^2) \left[ 1 + \sum_b 2 \frac{M_b^2}{M_1^2} (\chi_b^V)^2 \right] + \right. \\ &\quad \left. + (1 - (\chi_a^V)^2) \frac{2M_{\pm}^2}{M_a^2} N \right\} \\ N &= \sum_b \chi_b^V \chi_b^{\pm} - 1 + \text{Re} \left( \sum_b \chi_b^{H^+ W^-} \chi_b^{\pm} \frac{\chi_a^{H^+ W^-}}{\chi_a^V} \right). \end{aligned} \quad (36)$$

For  $a = 1$ , the first factor  $M_1^2/v$  is similar to what we would get in the SM when describing the triple Higgs vertex. This equation is used in [25] for estimating opportunity to observe sizable violation of this vertex from its SM value in the case of realization of SM-like scenario.

## VII. Notes about radiative corrections.

The standard calculation of the radiative corrections (RC) in the model is based on the parameters of Lagrangian which are RPa dependent. This RPa

ambiguity can be removed, for example, by using the renormalization procedure fixing parameters of the basic set. In the modern approach the calculation of any physical effect should be supplemented by calculation of renormalized values of masses and other parameters of basic set which should be taken into account in the data analysis<sup>2</sup>.

Besides, with anticipated low accuracy in the determination of parameters of Lagrangian from future data, the calculation of RC could help in the understanding of the model if only  $RC \gtrsim 10\%$ . These big RC can appear in the case of strong interaction in the Higgs sector (either direct or via  $t$ -quarks).

## VII. DISCUSSION

◊ We have found the minimal complete set of measurable quantities (named *observables*) which determines all parameters of the 2HDM Lagrangian – the *basic set of observables*. This set is naturally subdivided into two subsets.

The first subset contains masses of all Higgs bosons  $M_{1,2,3}$ ,  $M_{\pm}$ , vacuum expectation value of Higgs field  $v = 246$  GeV and the couplings  $\chi_a^V$  of any two (of three) chosen neutrals to the gauge bosons. Here we assume that  $h_1$  is the discovered Higgs boson with  $M_1 \approx 125$  GeV. The final equations also contain the couplings  $\chi_a^{H^+ W^-}$ , expressed via  $\chi_a^V$  with the aid of eq. (19). This subset determines explicitly all quadratic (mass) terms of potential (10).

The coefficients  $\Lambda_1, \Lambda_4, \Lambda_5, \Lambda_6$  of Lagrangian are expressed simply via observables of the first subset (21).

◊ The triple and quartic Higgs vertices of the potential (10) can be determined completely only if one supplements the parameters of the first subset by an additional information. In turn, to form the second subset, one need to use triple and quartic Higgs self-interactions. For this goal we use three triple couplings  $H^+ H^- h_a$  (quantities  $\chi_a^{\pm}$ ) and one quartic coupling  $g(H^+ H^- H^+ H^-)$ .

The parameters of the first subset plus three couplings  $\chi_a^{\pm}$  determine all triple Higgs couplings. The coefficients  $\Lambda_3, \Lambda_7$  of Lagrangian are expressed simply via these three couplings and observables of the first subset.

The description of quartic interactions of Higgs particles demands to add one more observable  $g(H^+ H^- H^+ H^-) = \Lambda_2/2$ .

• The obtained equations for parameters of Lagrangian in Higgs basis contain one irrelevant parameter: the RPh phase  $\rho$  related to a rephasing

<sup>2</sup> For example, in some particular variant of MSSM the value of triple Higgs coupling with RC looks essentially different from its tree form in the SM [22]. However, within the same approximation the using of the renormalized mass  $M_1$  makes the result close to the SM value [23].

freedom in the Higgs basis. In order to switch to another RPa basis, which could be more useful for some special reasons, one should use two parameters  $\tan\beta$  and  $\xi$ , which are determined by the RPa basis choice. Once these parameters are determined from problem-specific conditions, the transition to this RPa basis is performed with the aid of the back rotation  $\hat{\mathcal{F}}_{HB}^{-1}$  (9), as it was done in Sect. V. The final equations for parameters  $\lambda_i, m_{ij}^2$  are constructed from measurable quantities discussed above and RPa basis-choice parameters<sup>3</sup>  $\beta, \rho, \xi$ .

- The observables of the basic set are measurable quantities, independent of each other. The models with arbitrary values of these observable parameters can in principle be realized, provided that the positivity constraints like (32) are satisfied and the couplings  $\chi_a^V$  are not too large, in order not to violate the sum rule (20). In some special variants of 2HDM, additional relations between these parameters may appear (for example, in the CP conserving case  $\chi_3^V = \chi_3^\pm = 0$ ).

Our results open the door for the study of Higgs models in terms of measurable quantities alone. It allows to remove from the data analysis the widely spread intermediate stages with complex, often model-dependent, analysis of coefficients of Lagrangian.

- The principal possibility to determine all parameters of 2HDM from the (future) data meet strong practical

limitations (which can be hidden in other approaches). In the best case, it looks the problem for a very long time.

Indeed, the modern data on the Higgs boson couplings, the analysis of many particular models (see e.g. [20]), and the using of sum rules (20). (35) allow us to conclude that the discovery of new Higgs bosons  $h_{2,3}, H^\pm$  is a difficult problem for LHC and  $e^+e^-$  colliders [21].

If these  $h_{2,3}$  are discovered, the inaccuracies in the measuring of their masses and couplings are expected to be not small.

The measuring of triple and quartic interactions of Higgs bosons looks more difficult problem. So that it is natural to expect that these measurements will be made later and with bigger inaccuracy.

The general limitations for model, similar to the positivity constraint (32), contain parameters of the first and second subsets simultaneously. Thus, there are few chances that such restrictions can be verified in the near future.

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- [1] ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]]; CMS Collaboration, Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]]; M. Flechl, arXiv:1503.00632 [hep-ex]
  - [2] G. Belanger, B. Dumont, U. Ellwanger, J.F. Gunion, S. Kraml, JHEP **1302** (2013) 053 [arXiv:1212.5244 [hep-ph]]; B. Dumont, arXiv:1305.4635 [hep-ph]
  - [3] I. F. Ginzburg, M. Krawczyk and P. Osland, “Standard-model-like scenarios in the 2HDM and photon collider potential,” arXiv:hep-ph/0101331
  - [4] P.S.B. Dev, A. Pilaftsis, arXiv:1503.09140 [hep-ph]
  - [5] T.D. Lee, Phys. Rev. D **8** (1973) 1226; J.F. Gunion, H.E. Haber, G. Kane, S. Dawson, *The Higgs Hunter’s Guide* (Addison-Wesley, Reading, 1990); G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. **516**, 1 (2012) [arXiv:1106.0034 [hep-ph]].
  - [6] I. F. Ginzburg, I. P. Ivanov and K. A. Kanishev, Phys. Rev. D **81** (2010) 085031 [arXiv:0911.2383 [hep-ph]]; G. C. Dorsch, S. J. Huber and J. M. No, JHEP **1310**, 029 (2013) [arXiv:1305.6610 [hep-ph]].
  - [7] B. Dumont, J. F. Gunion, Y. Jiang and S. Kraml, Phys. Rev. D **90**, 035021 (2014) [arXiv:1405.3584 [hep-ph]].
  - [8] D. Lopez-Val and J. Sola, Phys. Rev. D **81**, 033003 (2010) [arXiv:0908.2898 [hep-ph]]; A. Barroso, P. M. Ferreira, I. P. Ivanov and R. Santos, JHEP **1306**, 045 (2013) [arXiv:1303.5098 [hep-ph]].
  - [9] J. F. Gunion, H. E. Haber, Phys. Rev. D **72** (2005) 095002 [hep-ph/0506227]; S. Davidson, H. E. Haber, Phys. Rev. D **72** (2005) 035004 [hep-ph/0504050]; G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Lett. B **614** (2005) 187 [hep-ph/0502118]; D.A. O’Neil, arXiv:0908.1363 [hep-ph]
  - [10] M. Maniatis, A. von Manteuffel, O. Nachtmann and F. Nagel, Eur. Phys. J. C **48**, 805 (2006) [hep-ph/0605184]; I. P. Ivanov, Phys. Lett. B **632**, 360 (2006) [hep-ph/0507132]; C. C. Nishi, Phys. Rev. D **74**, 036003 (2006) [E: D **76**, 119901 (2007)] [hep-ph/0605153]; I. P. Ivanov, Phys. Rev. D **75**, 035001 (2007) [E: D **76**, 039902 (2007)] [hep-ph/0609018]; Phys. Rev. D **77**, 015017 (2008) [arXiv:0710.3490]. P. M. Ferreira, H. E. Haber, M. Maniatis, O. Nachtmann and J. P. Silva, Int. J. Mod. Phys. A **26**, 769 (2011) [arXiv:1010.0935 [hep-ph]].
  - [11] D. Sokolowska, K. Kanishev, M. Krawczyk, *PoS CHARGED2008* 2008 016, arXiv:0812.0296
  - [12] I.F. Ginzburg, M. Krawczyk, Phys. Rev. D **72** (2005) 115013; hep-ph/0408011
  - [13] H. Georgi and D. V. Nanopoulos, Phys. Lett. B **82**, 95 (1979).
  - [14] I. F. Ginzburg and K. A. Kanishev, Phys. Rev. D **76** (2007) 095013 [arXiv:0704.3664 [hep-ph]].

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<sup>3</sup> Within Higgs sector all these parameters are non-physical since they describe only RPa freedom. The choice of f-selective RPa basis, in which the mass-generating Yukawa interaction of definite fermion  $f$  is written in the form (28), fixes value  $\beta = \beta_f$ . The parameter  $\beta$  accepts a meaning if it is the same for different fermions (or values  $\beta_f$  for different fermions are simply related to each other) – as in Models I or II.



- [15] M. Hashemi and I. Ahmed, arXiv:1401.4841 [hep-ph].
- [16] I. F. Ginzburg, JETP Lett. **99**, 742 (2014) [arXiv:1410.0873 [hep-ph]].
- [17] J. F. Gunion, H. E. Haber and J. Wudka, Phys. Rev. D **43**, 904 (1991).
- [18] A. Celis, V. Ilisie and A. Pich, JHEP **1307**, 053 (2013) [arXiv:1302.4022 [hep-ph]].
- [19] A. Pich and P. Tuzon, Phys. Rev. D **80**, 091702 (2009) [arXiv:0908.1554 [hep-ph]].
- [20] B. Holdom, M. Ratzlaff, arXiv:1412.1513 [hep-ph]; S. Kanemura, K. Tsumura, H. Yokoya, arXiv:1305.5424 [hep-ph]; N. Craig, J. Galloway, S. Thomas, arXiv:1305.2424 [hep-ph]; B. Dumont, J.F. Gunion, Y. Jiang, S. Kraml, arXiv:1405.3584 [hep-ph]; C. Lange, on behalf of the ATLAS and CMS, arXiv:1411.7279 [hep-ex]; D. Cline, X. Ding, J. Lederman, arXiv:1204.6700 [hep-ph]; J. Baglio, O. Eberhardt, U. Nierste, M. Wiebusch, arXiv:1403.1264 [hep-ph]
- [21] I.F. Ginzburg, arXiv:1502.07197 [hep-ph]
- [22] E. Asakawa, D. Harada, S. Kanemura, Y. Okada, K. Tsumura, *Phys.Rev.* **D82** (2010) 115002, arXiv:1009.4670 [hep-ph]
- [23] F. Boudjema, K. Kato, Y. Yasui, arXiv:0510184 [hep-ph]
- [24] B. Grzadkowski, O. M. Ogreid, P. Osland, arXiv:1504.06076 [hep-ph]
- [25] I.F. Ginzburg, arXiv:1505.01984 [hep-ph]